Capacity of Multiple Antenna Systems in Free Space and Above Perfect Ground

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***Abstract*—Multiple input–multiple output (MIMO) systems are a promising technique for wireless applications that require high data rates. In this letter, we theoretically study their capacity potential for propagation in free-space and over a ground surface. We investigate how this depends on the array geometry and the electric field polarization. Moreover, we validate our theoretical predictions with measurements above a nearly perfectly flat surface. The experiment shows good agreement with the theory for a large range of distances and deviates from it when secondary effects (roughness etc.) become more significant.**

***Index Terms*—Capacity, MIMO systems, propagation in free space.**

I. Introduction

IN RECENT years, a lot of attention has been drawn to systems with multiple element transmitter and receiver arrays, because they can achieve very high spectral efficiencies [1]. In this paper we investigate the potential of multiple input–multiple output (MIMO) systems under simple propagation conditions and we present relevant experimental results.

Assume a system with M transmitters and N receivers. Each transmitter m sends an independent data stream x_m with power E_x, so that the total transmitted power is $P_t = ME_x$. Let $\underline{x}$, $\underline{y}$ be the transmitted and the received signal vectors respectively. In the case of a flat-fading channel (no variation with frequency), the channel gain from transmitter j to receiver i is a scalar quantity, denoted T_{ij}. The transmitted and received vectors are related by the equation $\underline{y} = \mathbf{T}\underline{x} + \underline{n}$, where $\underline{n}$ is the receiver noise vector. The matrix $\mathbf{T}$ (channel transfer matrix) incorporates the channel transfer gains from each transmitter to each receiver. It is assumed that the noise at the receivers is Gaussian, of equal power σ^2 and its components are independent of each other, so that the noise autocorrelation matrix is $\mathbf{R}_{nn} = \sigma^2\mathbf{I}$ ($\mathbf{I}$: identity matrix). All the signals used in our formulation are discrete-time complex base-band, so the vectors $\underline{x}$, $\underline{y}$, $\underline{n}$ and the elements of the channel transfer matrix $\mathbf{T}$ are complex.

The generalized Shannon capacity for this channel is given by the formula

$$C = \log_2\left(\det\left(\mathbf{I} + \frac{E_x}{\sigma^2}\mathbf{T}\mathbf{T}^H\right)\right) \quad (1)$$

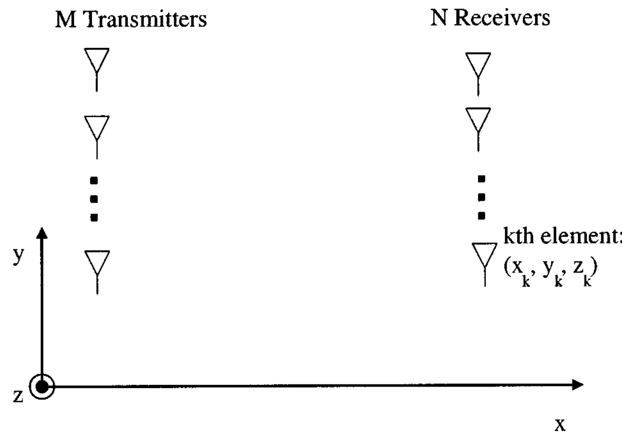

Fig. 1. Transmit and receive array geometry.

where $\mathbf{T}^H$ is the complex conjugate transpose of $\mathbf{T}$. We can define the following quantities:

Average channel gain g: $$g^2 = E[|T_{ij}|^2] \quad (2)$$

Average signal to noise ratio ρ: $$\rho = \frac{P_t}{\sigma^2}g^2 \quad (3)$$

Normalized channel transfer matrix $\mathbf{H}$: $$\mathbf{T} = g\mathbf{H}. \quad (4)$$

Similarly to the Shannon formula for the single transmitter/single receiver case, the channel capacity can be expressed in terms of the average signal to noise ratio ρ:

$$C = \log_2\left(\det\left(\mathbf{I} + \frac{\rho}{M}\mathbf{H}\mathbf{H}^H\right)\right). \quad (5)$$

In the initial theoretical studies [1] certain statistical properties have been assumed for the elements of $\mathbf{H}$. However there are simple propagation situations, such as in free space or above a perfect ground, where the channel transfer matrix is deterministic.

II. Capacity of Multiple Element Antenna Systems in Free Space

Let us assume arrays of isotropic antennas (M transmitters, N receivers), without any mutual coupling in free space, as shown in Fig. 1. Each element k is at a location (x_k, y_k, z_k), where all distances are normalized to the wavelength. The signal received at the kth element in the receiving array due to the excitation in the mth element of the transmitting array depends only on the distance r_{km} between them. The channel transfer coefficient T_{km} is given by

$$T_{km} = G\frac{e^{-j2r_{km}}}{r_{km}} \quad (6)$$

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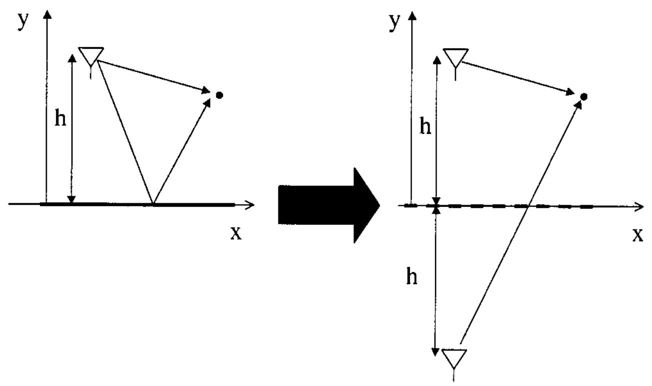

Fig. 2. Reflection off perfect ground.

where

$$r_{km} = \sqrt{(x_k - x_m)^2 + (y_k - y_m)^2 + (z_k - z_m)^2}. \quad (7)$$

The constant G incorporates the effect of element gain, wavelength, etc.

Given the channel transfer gain from each transmitter to each receiver, we can calculate the channel transfer matrix $\mathbf{T}$. From (5), the system capacity can be computed for any reference signal-to-noise ratio (SNR).

III. Capacity in the Presence of a Single Reflecting Surface

We use the geometrical configuration of Fig. 1, but we assume that at the xz plane there is a perfectly conducting ground plane.

Let us concentrate on a single transmitting antenna as in Fig. 2. Without loss of generality, let us assume that the original source is at the location $(0, y_0, 0)$. The method of images [2] is used to predict the electric field in the half-space $y \geq 0$. It utilizes the boundary condition on the ground plane, according to which the electric field has to be perpendicular to the perfectly conducting plane $(E_{\|} = 0)$.

The field in the half-space $y \leq 0$ is identically zero $(E = 0)$. The field at any location in the half-space $y \geq 0$ can be computed as the sum of the fields from the original source and from an image source at a location $(0, -y_0, 0)$. An alternate way to treat the signal from the image source is to consider it as signal from the original source that was reflected off the ground plane.

So at any location (x, y, z) $(y \geq 0)$ the field is given by

$$T = G\frac{e^{-j2\pi r}}{r} + RG'\frac{e^{-j2\pi r'}}{r'} \quad (8)$$

$$r = \sqrt{x^2 + (y_0 - y)^2 + z^2},$$
$$r' = \sqrt{x^2 + (y_0 + y)^2 + z^2}. \quad (9)$$

The factor G' again incorporates effects such as element gain, wavelength etc. For isotropic antennas, $G = G'$.

The factor R is the reflection coefficient off the ground plane

$$R = \begin{cases} 1, & \text{polarization } \| \text{ plane of incidence} \\ -1, & \text{polarization } \perp \text{ plane of incidence.} \end{cases} \quad (10)$$

If we assume arrays of isotropic antennas, without any mutual coupling, and of known polarization, we can repeat the above calculation for each transmitter-receiver pair and compute the channel transfer gains. From these we can formulate the channel transfer matrix $\mathbf{T}$ and compute the system capacity.

This analysis can be used as an approximation for the case of a dielectric or imperfectly conducting ground plane. In this case the reflection coefficient R depends on the dielectric/conductive properties of the boundary surface as well as the angle of incidence and the type of polarization.

Following the notation of [2]

$$R = \begin{cases} \dfrac{\varepsilon_r \cos\theta_i - \sqrt{\varepsilon_r - \sin^2\theta_i}}{\varepsilon_r \cos\theta_i + \sqrt{\varepsilon_r - \sin^2\theta_i}}, & \text{pol}\perp \text{ plane of incidence} \\ \dfrac{\sqrt{\varepsilon_r - \sin^2\theta_i} - \cos\theta_i}{\sqrt{\varepsilon_r - \sin^2\theta_i} + \cos\theta_i}, & \text{pol } \| \text{ plane of incidence} \end{cases} \quad (11)$$

ε_r is the relative dielectric constant. θ_i is the angle of incidence:

$$\cos\theta_i = \frac{\sqrt{x^2 + z^2}}{r'}. \quad (12)$$

For polarization parallel to the plane of incidence, there is a value of the incidence angle that makes the reflection coefficient equal to 0. This is called the Brewster angle and its value is given by

$$\theta_{\text{brewster}} = \tan^{-1}(\varepsilon_r). \quad (13)$$

At angles close to the Brewster angle, the reflected signal is greatly attenuated. There is no such angle for polarization perpendicular to the plane of incidence.

In the case of finite conductivity of the reflecting surface, the relative dielectric constant in (11) is replaced by the quantity

$$\varepsilon_r' = \varepsilon_r \left(1 - j\frac{\sigma}{\omega \varepsilon_r \varepsilon_0}\right). \quad (14)$$

where ε_0 is the permittivity of vacuum.

IV. Experimental Comparison

A. Description of the Experiment

In order to verify the theoretical predictions, we performed a set of measurements in the parking lot of the Bell Labs building in Crawford Hill, NJ.

The measurements were taken with a system of 12 transmitters and 15 receivers at a frequency of 1.95 GHz. The antennas used are flat arrays of folded cavity backed slot antenna elements mounted on $2' \times 2'$ panels, which have a hemispherical gain pattern. They were either vertically or horizontally polarized and arranged in alternate polarizations on 4×4 grids, separated by $\lambda/2$. Fig. 3 shows how the arrays look from the front (H/V: vertically/horizontally polarized elements). The system bandwidth was 30 kHz.

The prototype used for the measurement campaign processes data in bursts. Each burst consists of 100 symbols. Out of these, the first 20 are training symbols and are used for the measurement of the channel transfer matrix, using orthogonal training sequences as suggested in [3].

The measurements were performed in the parking lot, because it approximates the situation of an antenna array above a single reflecting surface. The parking lot is a flat asphalt-covered area in front of the building. In order to minimize reflections off

$\mathbf{V}_1$	$\mathbf{H}_6$	$\mathbf{V}_6$	$\mathbf{H}_5$
$\mathbf{H}_1$			$\mathbf{V}_5$
$\mathbf{V}_2$			$\mathbf{H}_4$
$\mathbf{H}_2$	$\mathbf{V}_3$	$\mathbf{H}_3$	$\mathbf{V}_4$

(a)

$\mathbf{H}_5$	$\mathbf{V}_6$	$\mathbf{H}_6$	$\mathbf{V}_1$
$\mathbf{V}_5$	$\mathbf{H}_8$		$\mathbf{H}_1$
$\mathbf{H}_4$	$\mathbf{V}_7$	$\mathbf{H}_7$	$\mathbf{V}_2$
$\mathbf{V}_4$	$\mathbf{H}_3$	$\mathbf{V}_3$	$\mathbf{H}_2$

(b)

Fig. 3. Array layout. (a) Transmitter side. (b) Receiver side.

the building, the hill and vehicles, the antennas were located at a distance of 20 ft from the side of the building, part of the parking lot was reserved for the experiment, and no cars were allowed in the vicinity.

The measurements were repeated for two array heights: the lowest element of the array was placed at a height of either 2 m (13.16λ) or 12 cm (0.94λ) off the ground. A limited set of transmitter-receiver separations, namely 1, 1.5, 2, 5, 10, and 15 m, were measured.

B. Experimental Results

Figs. 4 and 5 show how the capacity of the measured system (as calculated from the measured channel transfer matrices) matches the predicted capacity. We have also added the predictions for free space propagation. The results are shown for the reference SNR of 20 dB. The relative dielectric constant of the parking lot material (asphalt) was assumed to be $\varepsilon_r = 3$. Different reflection coefficients were calculated for each polarization. We assumed no cross-polarization coupling.

A 12×15 system with independent identically distributed Gaussian entries of the channel transfer matrix $\mathbf{T}$ at the same reference SNR would achieve a median capacity of 72 bps/Hz. The theory predicts capacities higher than 72 bps/Hz for short distances.

For some range of distances the theoretical predictions for free-space propagation are very close to the predictions that include the perfectly conducting ground plane. Indeed the reflected waves only become comparable to the direct waves, and therefore start to affect the capacity results, after the antennas have been sufficiently separated and the two paths have comparable lengths and strengths. The distance at which this occurs is larger for large antenna height as we observe from the comparison of Figs. 4 and 5.

The measurements agree fairly well with the predicted values for the capacity in the presence of a reflecting surface, especially at the small distances and for the large antenna height. As the separation between the transmitter and the receiver array increases, secondary effects become more significant:

The surface of the parking lot is not smooth. This causes back scattering, which would tend to increase the capacity of the measured system. This effect is more pronounced in the measurements at the low antenna height.

At larger distances, the reflections off the surrounding large surfaces (building, hillside, cars etc.) become more significant.

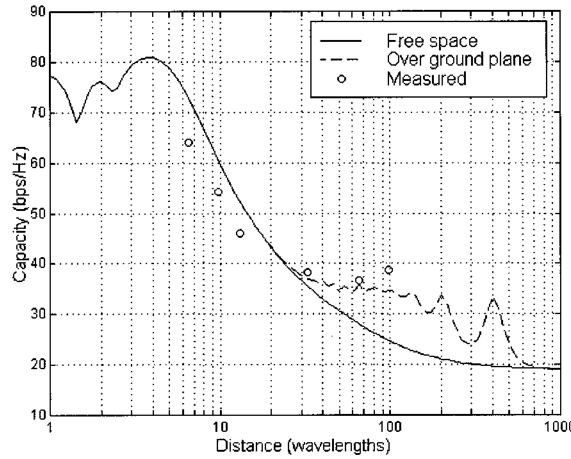

Fig. 4. Experimental comparison with the method of images for the 12×15 system at a height of 2 m and $\rho = 20$ dB.

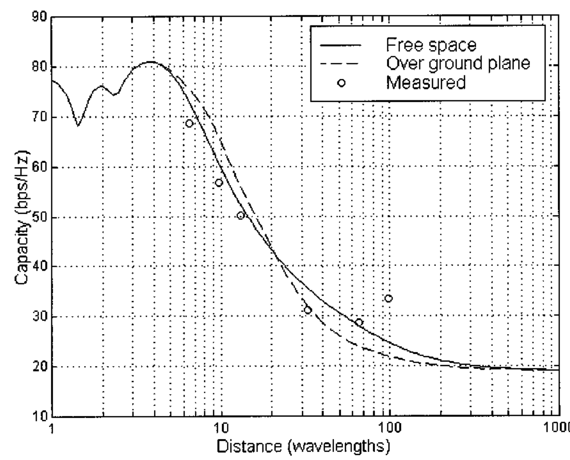

Fig. 5. Experimental comparison with the method of images for the 12×15 system at a height of 12 cm and $\rho = 20$ dB.

V. Conclusions

In this letter we have calculated the capacity of a MIMO system under simple propagation conditions, namely for propagation in free space and above a perfect ground. We validated our theoretical predictions with measurements above a nearly perfectly flat surface. The experiment shows good agreement with the theory for a large range of distances and deviates from it when secondary effects (roughness, etc.) become more significant.

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