Effect of Antenna Separation on the Capacity of BLAST in Correlated Channels

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***Abstract*—As the base station is usually placed above local clutter, the angular spectrum incident on the base is narrow, inducing correlation among base antenna signals, which reduces the capacity of a multiple transmit and receive antenna systems. In this work the general expression for link capacity is derived, when there is correlation among receive antennas and among transmit antennas. It is found that an antenna separation of 4 wavelengths between nearest neighbors in a linear base array of dually polarized antennas allows one to achieve 80% of the capacity attainable in the uncorrelated antenna case.**

I. Introduction

BLAST (Bell-labs LAyered Space–Time) is a communication technique for achieving very high spectral efficiencies in highly scattering environments using multiple transmit and receive antennas [1], [2]. These high spectral efficiencies are enabled by the fact that a scattering environment makes the signal from every individual transmitter appear highly uncorrelated at each of the receive antennas. As a result, the signal corresponding to every transmitter has a distinct spatial signature at the receiver. These different spatial signatures allow the receiver to effectively separate, with adequate signal processing, the data streams radiated—simultaneously and on the same frequency—by the different transmit antennas. In a sense, the scattering environment acts like a very large aperture that makes it possible for the receiver to resolve the individual transmitters and decode their data.

The high spectral efficiency is reduced if the signals arriving at the receivers are correlated. A narrow-band channel may be described in terms of a complex channel transfer matrix $\mathbf{H}$, whose entry h_{ij} corresponds to the response of the ith receiver to the signal sent by the jth transmitter. Maximum capacity is achieved when $\langle h_{ij} h_{kl}* \rangle = 0$ for $i \neq k$ and for $j \neq l$. It is the goal of this work to examine the impact of nonzero correlations, reported in the literature, on the capacity of BLAST.

II. Correlation Between Antenna Signals

The effect of fading correlation between antenna elements was explored by Shiu, *et al.* [6]. Signals received by two antennas are, in general, correlated [3], with the correlation coefficient depending both on the antenna separation (in wavelengths) and the angular spectrum of the incoming radio wave. Consider a linear array with regularly spaced antennas. The correlation coefficient of the signals received by antennas i and j, separated by distance d_{ij}, and illuminated by an angular spectrum $p(\alpha)$ is [3].

$$\rho_{ij} = \int_0^{2\pi} e^{ikd_{ij}\cos(\alpha-\phi)} p(\alpha)\, d\alpha, \qquad \int_0^{2\pi} p(\alpha)\, d\alpha = 1 \tag{1}$$

where α is the azimuth angle of incidence of each plane wave; $k = \omega/c$ is the wavenumber at the angular frequency ω; and ϕ is the angle of orientation of the linear array, set to be 90° for the broadside array considered here. In the case of uniform illumination $p(\alpha) = 1/2\pi$ and $\rho_{ij} = J_0(kd_{ij})$, where J_0 is the zeroth order Bessel function. To achieve near complete decorrelation in this case, the antenna elements should be spaced about $\lambda/2$ apart. To determine the effect of antenna correlation on the capacity of BLAST, we assume that the base station antennas are placed at the same height, e.g. rooftop, as the current cellular antennas. It has been reported by Chu and Greenstein [4] that the angular spread of signals arriving at PCS base stations varies with distance, and is about 2° at 1 km. This was deduced from antenna correlation measurements reported by Rhee and Zysman [5]. Two distributions $p(\alpha)$ with the same rms spread of σ will be used in this study: a uniform distribution of width $2\sigma\sqrt{3}$ and a Gaussian distribution with a standard deviation of σ. The value of σ will be set at 2°, as the urban area with cell radii of less than 1 km is of primary interest to wideband wireless communication services. Fig. 1 illustrates the dependence of the correlation coefficient ρ on the antenna separation for both uniform and Gaussian shaped angular spectrum. Note that the Gaussian angular spectrum results in a correlation coefficient, which is monotonically decreasing with antenna separation.

III. Capacity of BLAST in Correlated Channels

The generalized Shannon bound on capacity has been presented by Foschini and Gans [1] as

$$C = \log_2 \left(\det \left(I + \frac{\rho}{n_T} HH^* \right) \right) \text{ (bps/Hz)} \tag{2}$$

where ρ is the average received signal to noise ratio (SNR), assuming identically distributed noise at each receiver, transmit power equally distributed among transmit antennas; $\mathbf{H}$ is the normalized channel transfer matrix; and $*$ corresponds to conjugate transpose.

The entry h_{ij} of the channel matrix represents the normalized channel transfer function, evaluated at the frequency of operation, between transmit antenna j and receive antenna i. In statistical modeling, these entries are usually chosen from a distribution of zero-mean, unity variance complex Gaussian processes,

Manuscript received April 7, 2000. The associate editor coordinating the review of this letter and approving it for publication was Prof. M. Tsatsanis.

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Publisher Item Identifier S 1089-7798(00)10981-0.

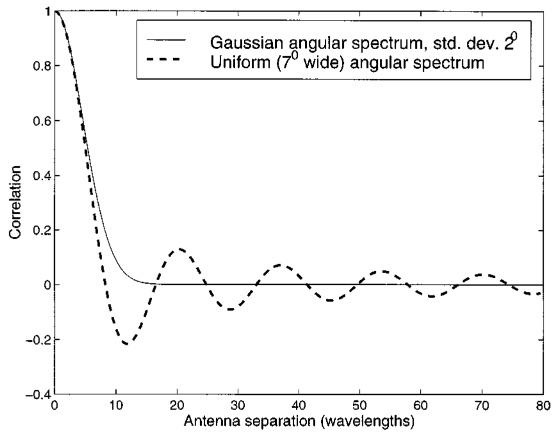

Fig. 1. Correlation coefficient between antenna elements for different shaped angular spectra with $\sigma = 2°$.

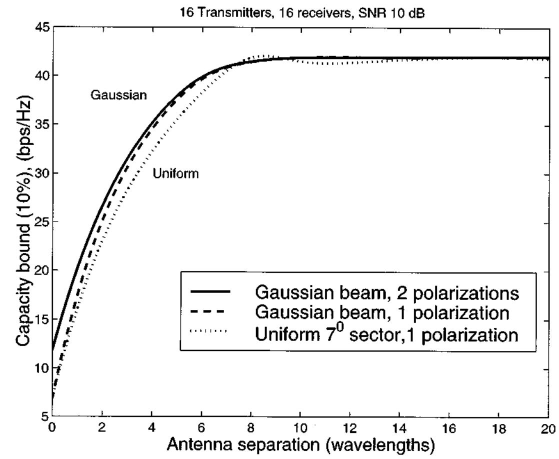

Fig. 2. 10% Capacity bound versus antenna separations for two shapes of angular spectrum and with dual and single polarized antennas. Both spectra have a $\sigma = 2°$. $\mathrm{SNR} = 10$ dB.

which are independent, and thus, uncorrelated. The correlation is now introduced.

The most general type of correlation between the various elements of $\mathbf{H}$ may be represented as a four dimensional tensor $\mathbf{K}$, so that a general matrix with correlated entries may be obtained from the matrix $\mathbf{H}$ which has uncorrelated complex Gaussian entries:

$$H^c = KH \tag{3}$$

where each entry is $h_{ij} = \sum_{p,q} K_{ijpq} h_{pq}$.

Here a special case of the above relationship is considered, i.e., modeling the correlation among receiver and transmitter array elements independently from one another. The underlying justification for this approach is to assume that only immediate surroundings of the antenna array impose the correlation between array elements and have no impact on correlations observed between the elements of the array at the other end of the link [7]. One way to effect this type of antenna signal correlation is to post-multiply the channel transfer matrix $\mathbf{H}$ described above by a transformation matrix $\mathbf{K_T}$, and pre-multiply by a transformation $\mathbf{K_R}$ which produces a new channel transfer matrix whose entries are now correlated along any row and any column

$$H_1 = K_R H K_T \tag{4}$$

The capacity now becomes

$$\begin{aligned} C &= \log_2\left(\det\left(I + \frac{\rho}{n_T} K_R H K_T K_T^* H^* K_R^*\right)\right) \\ &= \log_2\left(\det\left(I + \frac{\rho}{n_T} \Phi_R H \Phi_T H^*\right)\right) \text{ (bps/Hz)} \end{aligned} \tag{5}$$

using the identity

$$\det(I + AB) = \det(I + BA) \tag{6}$$

where $\Phi_T = \boldsymbol{K_T K_T}*$ and $\Phi_R = \boldsymbol{K_R^* K_R}$ are the covariance matrices of the transmit and receive antenna arrays, respectively.

The entries of the covariance matrices, corresponding to co-polarized antennas, are given by the correlation coefficients (1). It is assumed here that there is no correlation between antennas of orthogonal polarization. For the dual-polarization case, the total covariance matrix thus takes on a block diagonal form

$$\Phi_{\text{total}} = \begin{pmatrix} \Phi & 0 \\ 0 & \Phi \end{pmatrix} \tag{7}$$

Given M antennas in the array, the total covariance matrix Φ_{total} has the dimension $M \times M$, while the submatrices Φ have the dimension $M/2 \times M/2$.

In this letter the link capacity of a system with 16 transmitters and 16 receivers at an average SNR of 10 dB is examined, under the assumption that the signals from different base antennas are zero-mean complex Gaussian random variables with correlation given by (1). The effect of the nonzero mean was considered in [8] by modeling the signal statistics as Rician with a nonzero K-factor. Remote antenna array elements are assumed to be uncorrelated, which is appropriate for the case of an array immersed in clutter with a separation of a half wavelength between elements [3]. Using (7) to impose correlation on random H matrices, a cumulative distribution function (cdf) of capacity has been generated. The tenth percentile of the capacity is shown in Fig. 2 for various spectral shapes and number of polarizations.

Note that when the shape of the angular spectrum at the transmitter is Gaussian, the maximum capacity bound is reached at a slightly smaller antenna separation than in the case of the uniform sector. For example, a separation of four wavelengths allows for achieving 80% of the maximum capacity under the Gaussian spectrum assumption. Full capacity bound is realizable at 10 wavelengths. The antenna separations relevant here are between co-polarized antennas. The size of the antenna array may be reduced by the use of dually polarized antennas. In Fig. 2, the array performance using an array with half of its antennas polarized orthogonally to the other half is shown. It may be seen that the dually polarized array achieves slightly higher capacity. The case when antenna separation approaches 0 is equivalent to the angular spectrum width at the arrays collapsing to 0. In that case, the dually polarized array performs significantly better as there are still 2 degrees of freedom corresponding to the signals arriving in two different polarizations.

IV. Conclusions

The correlation between signals at base station antennas has been determined for different antenna separations. The result is used to derive link capacity, when there is correlation between receive antennas and between transmit antennas. It is found that for a 16×16 BLAST system at 10 dB SNR, the antenna separation of 4 wavelengths between nearest neighbors in a linear array allows one to achieve 36 bps/Hz as compared to 42 bps/Hz for the uncorrelated antenna case. Here it is assumed that there is correlation only at the base station and remote antennas are uncorrelated because the remote is in a cluttered environment. Use of dual polarization antennas has also been explored and found to allow a reduction of the base station array size in half, while slightly improving the capacity.

Acknowledgment

The authors would like to thank M. Gans, N. Amitay, J. Foschini and R. Valenzuela for many helpful discussions.

References

[1] G. J. Foschini and M. J. Gans, "On limits of wireless communications in a fading environment when using multiple antennas," *Wireless Pers. Commun.*, vol. 6, no. 3, p. 311, Mar. 1998.

[2] G. J. Foschini, "Layered space–time architecture for wireless communication in a fading environment when using multiple antennas," *Bell Labs Tech. J.*, vol. 1, no. 2, pp. 41–59, 1996.

[3] W.C. Jakes, Ed., *Microwave Mobile Communications*. New York, NY: Wiley, 1974.

[4] T.-S. Chu and L. J. Greenstein, "A semiempirical representation of antenna diversity gain at cellular and PCS base stations," *IEEE Trans. Commun.*, vol. 45, pp. 644–646, June 1997.

[5] S. B. Rhee and G. I. Zysman, "Result of suburban base station diversity measurements in the UHF band," *IEEE Trans. Communications*, vol. 22, pp. 1630–1636, Oct. 1974.

[6] D.-S. Shiu, G. J. Foschini, M. J. Gans, and J. M. Kahn, "Fading correlation and its effect on the capacity of multi-element antenna systems," in ICUPC 98, Florence, Italy, 1998.

[7] A. Moustakas, H. Baranger, L. Balents, A. Sengupta, and S. Simon, "Communication through a diffusive medium: Coherence and Capacity," *Science*, vol. 287, pp. 287–290, Jan 2000.

[8] F. R. Farrokhi, G. J. Foschini, A. Lozano, and R. A. Valenzuela, "Link-optimal space–time processing with multiple transmit and receive antennas," *IEEE Commun. Lett.*, to be published.